DSP notes 1

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1 DTFT Time Reversal Property: Variable Substitution Method

1.1 Core Concept: Variable Substitution

When we substitute variables in an equation, we're essentially relabeling our terms without changing the mathematical result. Think of it like renaming items in a list—the content stays the same, just the labels change.

1.2 DTFT Time Reversal Proof (Step-by-Step)

Starting Point: We want to find the DTFT of a time-reversed signal $x[-n]$:

$$DTFT(x[-n]) = \sum_{n=-\infty}^{\infty} x[-n]e^{-j\omega n} \tag{1}$$

Step 1: Variable Substitution Let $k = -n$, which means $n = -k$
Step 2: Rewrite the Sum When $n = -k$, our sum becomes:

$$\sum_{k=-\infty}^{\infty} x[k]e^{-j\omega(-k)} = \sum_{k=-\infty}^{\infty} x[k]e^{j\omega k} \tag{2}$$

Step 3: Final Result This equals:

$$DTFT(x[-n]) = \sum_{k=-\infty}^{\infty} x[k]e^{j\omega k} = X(e^{j\omega})|_{\omega\to-\omega} \tag{3}$$

where $X(e^{j\omega})$ is the DTFT of $x[n]$.

1.3 Key Insights

1. **Why the bounds stay the same:** When we substitute $k = -n$, as n goes from $-\infty$ to $+\infty$, k also goes from $+\infty$ to $-\infty$, which is the same as $-\infty$ to $+\infty$ (since we're summing over all integers).

2. **The sign flip:** The negative sign in the exponent flips to positive because we replaced n with $-k$.

3. **Physical meaning:** Time-reversing a signal in the time domain corresponds to frequency reversal (replacing ω with $-\omega$) in the frequency domain.

1.4 Time Reversal Property

The proof shows that:

$$DTFT(x[-n]) = X(e^{-j\omega}) \tag{4}$$

This is the **time reversal property** of the DTFT, which states that time-reversing a signal corresponds to frequency reversal in its DTFT.

1.5 Summary

The key principle is that variable substitution preserves the mathematical equality while allowing us to manipulate the form of our equations. In this case:

- Original signal: $x[n] \leftrightarrow X(e^{j\omega})$
- Time-reversed signal: $x[-n] \leftrightarrow X(e^{-j\omega})$